



To Be or Not to Be, That is the Question

Radial Liquid Films and their Stability or Instability due to Film Thinning

In the course of their experimental investigation on free liquid films by means of the so-called chromatic confocal measurement method, researchers of the Institute of Mechanical Process Engineering (IMVT) at the University of Stuttgart became aware of an instability pertaining to radially expanding and sinusoidally forced free liquid films, which was shown to exist based on analytical considerations by a research group at the Indian Institute of Technology (IIT) [1]. In contrast to the well-known capillary-driven instability found on radial films (and documented by G. I. Taylor in 1959); this presumably “newly discovered” instability originated from inertia-based higher-order terms (in the relative thickness parameter ε) within the linearized film equations, which were found to result film thinning in radial direction.

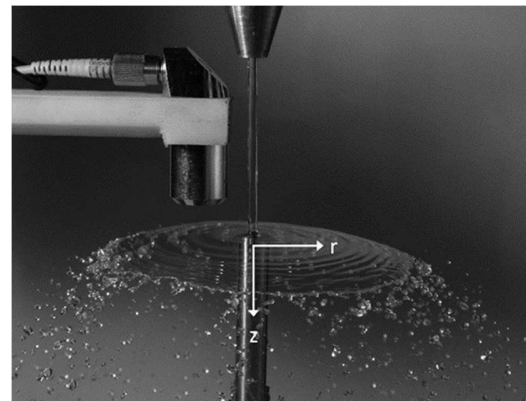
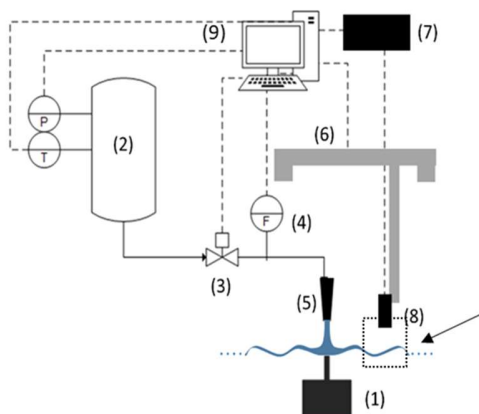


Figure 1. Left: Schematic set-up for the experimental investigation of free radially expanding liquid films, resulting from the impact of a vertical liquid jet (blue) on a vertically oscillating impact cylinder (Pos. 1). Right: Photographic image of radially expanding film with imposed sinuous disturbances (the chromatic confocal sensor is located just left of the impacting liquid jet).

Despite extensive experimental investigations (see Fig. 1), evidence of the prescribed inertia-based film instability could not be found [2]. Accordingly, the higher-order perturbation expansion published by the IIT group was reviewed in detail and carried out anew. It could be shown that the original derivation process was mathematically inconsistent. The correct and consistent derivation of the linear perturbation equations for the radially expanding and sinusoidally excited film, accurate to second-order in the perturbation parameter ε , shows that an inertia-based instability as a consequence of film thinning does not exist [3] (see Figure 2). Accordingly, prior studies discussing the previously asserted (inertia-based) instability based on film thinning, have to be reevaluated.

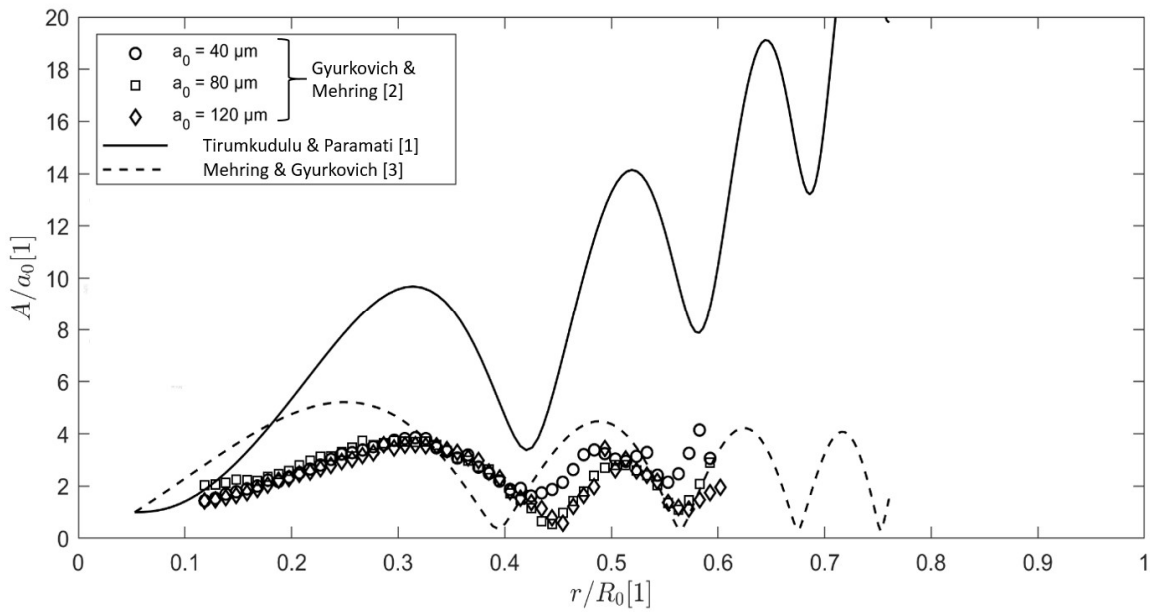


Figure 2. Symbols: Measurement data taken from [2] for the envelope of the film disturbance as function of dimensionless radius for different disturbance amplitudes $a_0 = \{40, 80, 120\} \mu\text{m}$ of the oscillating impact cylinder at Weber number $We_d = 540$ (based on diameter and mean velocity of the impacting liquid jet) and oscillation frequency $f_0 = 80 \text{ Hz}$ of the impact cylinder. Also shown, analytical solutions according to the model equations from [1] (solid line) and the corrected model equations from [3] (dashed line).

[1] M. S. Tirumkudulu and M. Paramati, "Stability of a moving radial liquid sheet: Time-dependent equations," *Phys. Fluids* 25(10), 102107 (2013).

[2] A. Gyurkovich and C. Mehring, "Measurements of sinuous waves on mechanically forced radially expanding free planar liquid sheets using chromatic confocal measurements", *Atom. Spray*, 33 (6): 19-44 (2023).

[3] C. Mehring and A. Gyurkovich, "Comment on 'Stability of a moving radial liquid sheet: Time-dependent equations' [*Phys. Fluids* 25, 102107 (2013)]", *Phys. Fluids*, 35, 099101 (2023).